Online note on the decibel

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Dr. Samuel Ritchie

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Voltage Ratio's

In the PW article we only looked at power ratios but it is not uncommon for voltage and current ratios to be used to compute dB gain or loss, provided that input and output impedances are taken into account. To convert a linear voltage ratio to a voltage ratio in dB is:

$$Voltage\ Ratio\ (in\ dB) = 20 \cdot \log_{10}(linear\ voltage\ ratio)$$

The basis of this equation is $Ratio\ (in\ dB) = 10 \cdot \log_{10} \frac{P_o}{P_i}$ and we also know that $P = \frac{V^2}{R}$ so:

Power ratio

$$Ratio\ (in\ dB) = 10 \cdot \log_{10} \frac{P_o}{P_i}$$

$P_o = \frac{{V_o}^2}{R_o}$ and $P_i = \frac{{V_i}^2}{R_i}$

$$Ratio\ (in\ dB) = 10 \cdot \log_{10} \frac{\frac{{V_o}^2}{R_o}}{\frac{{V_i}^2}{R_i}}$$

Rearrange

If R_o does not equals R_i then use this equation.

$$Ratio\ (in\ dB) = 10 \cdot \log_{10} \frac{{V_o}^2}{R_o} \cdot \frac{R_i}{{V_i}^2}$$

If $R_o = R_i$ they cancel

$$Ratio\ (in\ dB) = 10 \cdot \log_{10} \frac{{V_o}^2}{{V_i}^2}$$

Simplify

$$Ratio\ (in\ dB) = 10 \cdot log_{10} \left(\frac{V_o}{V_i}\right)^2$$

Apply rule 3

$$Ratio\ (in\ dB) = 20 \cdot log_{10} \left(\frac{V_o}{V_i}\right)$$

An Example – this one comes from page 592, example 7 from Basic Mathematics for Electronics, 5th edition, Cooke, Adams and Dell, ISBN 0-07-012514-7

You have an audio amplifier with an input resistance of 200 ohms and an output resistance of 6400 ohms. When 0.5V is applied on the input a voltage of 400V appears on the output.

1. *What is the output power in Watts?*
2. *What is the amplifier gain in dB?*

1. $P_{out} = P_o = \frac{{V_o}^2}{R_o} = \frac{(400)^2}{6400} = 25\,Watts$

2. $P_{in} = P_i = \frac{V_i{}^2}{R_i} = \frac{(0.5)^2}{200} = 0.00125\ W\ or\ 1.25mW$

$$Gain = \frac{P_o}{P_i} = \frac{25}{0.00125} = 20\ 000\ times$$

$$Gain\ (dB) = 10\ Log\ 20000 = 43\ dB$$

Or by using voltages:

$$Ratio\ (in\ dB) = 10 \cdot \log_{10} \frac{V_o{}^2}{R_o} \cdot \frac{R_i}{V_i{}^2}$$

$$Gain\ (dB) = 10 \cdot \log_{10} \frac{(400)^2}{6400} \cdot \frac{200}{(0.5)^2} = 43\ dB$$

Current Ratio's

To convert a linear current ratio to a current ratio in dB is:

$$Current\ Ratio\ (in\ dB) = 20 \cdot \log_{10}(linear\ current\ ratio)$$

The basis of this equation is $Ratio\ (in\ dB) = 10 \cdot \log_{10} \frac{P_o}{P_i}$ and we also know that $P = I^2 \cdot R$ so:

Power ratio	$Ratio\ (in\ dB) = 10 \cdot \log_{10} \frac{P_o}{P_i}$
$P_o = I_o{}^2 \cdot R_o$ and $P_i = I_i{}^2 \cdot R_i$ If R_o does not equal R_i then use this equation	$Ratio\ (in\ dB) = 10 \cdot \log_{10} \frac{I_o{}^2 \cdot R_o}{I_i{}^2 \cdot R_i}$
If $R_o = R_i$ they cancel	$Ratio\ (in\ dB) = 10 \cdot \log_{10} \frac{I_o{}^2}{I_i{}^2}$
Simplify	$Ratio\ (in\ dB) = 10 \cdot log_{10} \left(\frac{I_o}{I_i}\right)^2$
Apply rule 3	$Ratio\ (in\ dB) = 20 \cdot log_{10} \left(\frac{I_o}{I_i}\right)$

Common dB definitions

Here is a non-definitive list of frequently used reference values[1].

dB_m	dB value of power where 0 dB_m = 1 milliwatt.
dB_w	dB value of power where 0 dB_w = 1 Watt.
dB_i	dB value of antenna gain where 0 dB_i = 1
dB_{sm}	dB value of Area where 0 dB_{sm} = 1 square meter (common in antenna area and radar cross section calculations)
$dB_{\mu V}$	dB value of voltage, expressed in microvolts where 0 $dB_{\mu V} = 1\mu V$
$dB_{m/Hz}$	The amount of energy measured in dB_m, in a 1 Hz bandwidth.
dB_{W/m^2}	Measurement of Power Flux Density.
$dB_{\mu V/m}$	Measurement of Electric Field Strength.
dB_i	Value of antenna gain relative to an isotropic antenna.
dB_d	Value of antenna gain relative to the gain of a reference dipole antenna.

Table 1. Non-definitive list of common reference values.

[1] A short guide to many of the measurements made in deciBels is contained in section 5 of "Everything you ever wanted to know about decibels but were afraid to ask" www.rohde-schwarz.de/file_5613/1MA98_4E.pdf

Q & A TO TEST YOUR SKILLS

Answers in (   ).

3. *How many dB greater than 4 watts is 64 watts?   (+ 12 dB)*
4. *How many dB greater than 10 watts is 1000 watts? (+ 20 dB)*
5. *How many dB greater than 1 watt is 80 watts?   (+19 dB)*
6. *How many dB less than 1 watt is 1 milliwatt (0.001 watts)?   (-30 dB or 30 dB less)*
7. *How many dB less than 1 watt is 4 milliwatts (0.004 watts)? (-24 dB or 24 dB less)*
8. *How many dB greater than 63 watts is 700 watts? (+10.46 dB or 10.46 dB greater)*
9. *How many dB greater than 3,000 watts is 10,000 watts? (+5.23 dB or 5.23 dB greater)*
10. *How many dB greater than 2 volts is 10 volts? (+14 dB or 14 dB greater)*
11. *How many dB greater than 1 volt is 100 volts? (+40 dB or 40 dB greater)*
12. *How many dB greater than 2 volts is 63 volts? (+30 dB or 30 dB greater)*
13. *If you have a signal generator with an output power range of -140 dB_m to +20dB_m and you connected a power meter reading in Watts, assuming the input and output impedances are equal what range does the Watt meter need to cover? { 0.01 fW (femto Watt) to 0.1 W} {0.01fW = 10aW (attoWatt)}*
14. *What is the level of a power flux density of 5W/m² ? {7 dBW/m² }*
15. *Express a voltage of 7 μV as a level in $dB_{\mu V}$ { 16.9 $dB_{\mu V}$}*
16. *What the power of a power level of -3 dB_W? {0.5 W or 500 mW}*
17. *What is the voltage of a voltage level of 120 $dB_{\mu V}$? { 1V }*

Working with Logs - all the mathematics you need to know

Here are the 7 rules of using logs:

Rule 1: $log_a(x \cdot y) = log_a x + log_a y$

Rule 2: $log_a\left(\frac{x}{y}\right) = log_a x - log_a y$

Rule 3: $log_a(x^n) = n \cdot log_a x$

Rule 4: $log_a a = 1$

Rule 5: $log_a 1 = 0 \quad for\ any\ base\ a.$

Rule 6: $a^{log_a x} = x$

Rule 7: $a^o = 1$

Four questions that show some of the rules in action.

1. Show that if $x = a^t$ then $t = \log_a x$

	$x = a^t$
take the log of both sides to base a:	$\log_a x = \log_a a^t$
apply rule 3 to RHS	$\log_a x = t \cdot \log_a a$
apply rule 4 to RHS	$\log_a x = t \cdot 1$
re-order and done	$t = \log_a x$

2. *Show that if* $t = log_a x$ *then* $x = a^t$

	$t = log_a x$
raise both sides to the base a	$a^t = a^{log_a x}$
apply rule 6 to RHS	$a^t = x$
re-order and done	$x = a^t$

3. Show that rule 7 can be derived from rule 5

	$log_a 1 = 0 \quad for\ any\ base\ a.$
raise both sides to the base a	$a^{log_a 1} = a^0$
apply rule 6 to LHS	$1 = a^0$
re-order and done	$a^0 = 1$

4. Show that $log_a \left(\frac{1}{x}\right) = -\, log_a x$

	$log_a \left(\frac{1}{x}\right)$
apply rule 2	$\log_a 1 - \log_a x$
apply rule 5	$0 - \log_a x$
done	$-\, log_a x$

Examples of working with Logs

Split $\log_2 8x^2$ as far as possible:

	$log_2 8x^2$
Apply rule 1	$log_2 8 + \log_2 x^2$
8 = 2 × 2 × 2 = 2³	$log_2 2^3 + log_2 x^2$
Apply rule 3	$3 \log_2 2 + 2 \log_2 x$
Apply rule 4	$3 \times 1 + 2\, log_2 x$
Answer	$3 + 2\, log_2 x$

Split $log_2 \left(\frac{3x^2}{y^3}\right)$ *as far as possible:*

	$log_2 \left(\frac{3x^2}{y^3}\right)$
Apply rule 2	$log_2 3x^2 - \log_2 y^3$
Apply Rule 1	$log_2 3 + log_2 x^2 - log_2 y^3$

Apply rule 3 and answer $log_2 3 + 2\log_2 x - 3\log_2 y$

Combine $\log_2 3 + 4log_2 x$ *as far as possible*

$$log_2 3 + 4log_2 x$$

Apply rule 3 $log_2 3 + \log_2 x^4$

Apply rule 1 and answer $log_2 3x^4$

Combine $log_{10}(x^2 + 1) - log_{10}(x^2 - 1)$ *as far as possible*

$$log_{10}(x^2 + 1) - log_{10}(x^2 - 1)$$

Apply rule 2 and answer $\log_{10}\left(\frac{x^2 + 1}{x^2 - 1}\right)$

You cannot split what is inside the brackets – we have no rule for doing that.

There are some more practice Q & A's in annex 1.

To make maximum gain out of using logs it is common to put numbers into dB form as soon as possible in the calculation process and either leave the answer as dB form or convert to linear form as late as possible in the calculation process.

In many textbooks the convention is to drop the base of the logarithm, just to declutter the equations. So when you see $\log x$ what is meant is $\log_{10} x$. In other papers and text books the authors go one further and refer to $\lg x$ when they mean $\log_{10} x$.

Real World Example – turning linear equations into log form.

Here is an equation in linear form, where d = distance in meters, f = frequency in Hz, c = speed of light and FSL = Free Space Loss:

Basic equation	$FSL = \left(\frac{4 \cdot \pi \cdot d}{\frac{c}{f}}\right)^2$
Rearrange	$\sqrt{FSL} = \frac{4 \cdot \pi \cdot d}{1} \cdot \frac{f}{c}$
Gather constants and separate variables	$FSL = \left(\frac{4 \cdot \pi}{c}\right)^2 \cdot d^2 \cdot f^2 \quad (dimensionless - a\ ratio)$
Convert the equation from linear form to dB	$FSL_{\boldsymbol{dB}} = 10 \cdot log_{10}\left(\frac{4 \cdot \pi}{c} \cdot d \cdot f\right)^2 \quad (dB)$
Apply rule 3	$FSL_{\boldsymbol{dB}} = 20 \cdot log_{10}\left(\frac{4 \cdot \pi}{c} \cdot d \cdot f\right) \quad (dB)$
Apply rule 1 *(Note here I could have split out the constants but chose to keep them together)*	$FSL_{\boldsymbol{dB}} = 20 \cdot log\left(\frac{4 \cdot \pi}{c}\right) + 20 \cdot log(d) + 20 \cdot log(f) \quad (dB)$
Taking c = 3 × 10⁸ (m/s⁻¹)	$FSL_{dB} = 20 \cdot log\left(\frac{4 \cdot \pi}{3 \times 10^8}\right) + 20 \cdot log(d) + 20 \cdot log(f)$
Calculate the first term as it is a constant.	$FSL_{dB} = -147.6 + 20 \cdot log(d) + 20 \cdot log(f) \quad (dB)$

You may come across a few FSL formula that look similar in form to what we have above but where the constant (-147.6) is a different value. If you look carefully you will see that either distance is in km and not meters and / or frequency is in MHz or even GHz.

To show how this is done let us change the value entered for distance from meters to kilometers.

$$Knowing\ that\ 1\ km = 1000\ m$$

$$\begin{aligned} Conversion\ factor &= 20 \cdot \log_{10} 1000 \\ &= 60\ \ dB \end{aligned}$$

So:

$$FSL_{dB} = -147.6 + 20 \cdot \log(d) + 20 \cdot \log(f) + \mathbf{60}$$

$$FSL_{dB} = -87.6 + 20 \cdot \log(d) + 20 \cdot \log(f) \qquad (dB)$$

Where d = distance in kilometers (km)

To make the equation even more useful lets change the frequency entered from Hertz to Megahertz (MHz).

$$Knowing\ that\ 1\ MHz = 1\,000\,000\ Hz$$

$$\begin{aligned} Conversion\ factor &= 20 \cdot \log_{10} 1\,000\,000 \\ &= 120\ dB \end{aligned}$$

So

$$FSL_{dB} = -87.6 + 20 \cdot \log(d) + 20 \cdot \log(f) + \mathbf{120}$$

$$FSL_{dB} = 32.4 + 20 \cdot \log(d) + 20 \cdot \log(f) \quad (dB)$$

Where d= distance in kilometers (km) and f = frequency in Megahertz (MHz).

Show that FSL has no dimensions:

4π is a constant and has no dimensions so I will use 1

F = Hz = 1/t =1/s

C = m/s

D = m

$$FSL = \left(\frac{4 \cdot \pi}{c}\right)^2 \cdot d^2 \cdot f^2$$

$$\text{FSL} = \frac{4 \cdot \pi}{c} \cdot \frac{4 \cdot \pi}{c} \cdot \frac{d}{1} \cdot \frac{d}{1} \cdot \frac{f}{1} \cdot \frac{f}{1}$$

$$\text{FSL} = \frac{1}{\frac{m}{s}} \cdot \frac{1}{\frac{m}{s}} \cdot \frac{m}{1} \cdot \frac{m}{1} \cdot \frac{Hz}{1} \cdot \frac{Hz}{1}$$

$$\text{FSL} = \frac{1}{1} \cdot \frac{s}{m} \cdot \frac{1}{1} \cdot \frac{s}{m} \cdot \frac{m}{1} \cdot \frac{m}{1} \cdot \frac{1}{s} \cdot \frac{1}{s}$$

$$\text{FSL} = \text{dimentionless}$$

And now knowing that the calculation of Free Space Loss has no unit (it is dimensionless), when we turn it into a logarithm we can just use dB.

Annex 1: Working with logs – practice Q & A

Split the following expressions as much as possible – answers in { }.

$\log_3 3x$	$\{1 + \log_3 x\}$
$\log_3 27x^2$	$\{3 + 2\log_3 x\}$
$\log_5 \frac{x}{y}$	$\{\log_5 x - \log_5 y\}$
$\log_7 \frac{x^2}{a^2}$	$\{2\log_7 x - 2\log_7 a\}$
$log_{10}(ax^n)$	$\{\log_{10} a + n\log_{10} x\}$
$log_3 9a^x$	$\{2 + x\log_3 a\}$
$log_3(2x + 3y)$	{no expansion is possible}

Combine the following expressions as far as possible – answers in { }.

$\log_{10} x + \log_{10}(x-1)$	$\{\log_{10}(x^2 - x)\}$
$2\log_{10} x - log_{10} y$	$\{\log_{10}\left(\frac{x^2}{y}\right)\}$
$log_{10}(x+1) - log_{10}(x-1)$	$\{log_{10}\left(\frac{x+1}{x-1}\right)\}$
$3log_{10} x + 2\,log_{10} y$	$\{\log_{10}(x^3 \cdot y^2\}$

Annex 2: A more advanced problem

The following figure is copied from appendix A of an academic journal article[2]:

APPENDIX A

In this section, we summarize the relationship between the EIRP of a transmit antenna and the field strength $\mathcal{E}$ at the receiver antenna location due to its importance through this paper. Recall that the power received by an isotropic antenna $P_r = P_d A_e$, in watts, where A_e is the effective area (aperture) of the receive antenna, in m^2; $P_d = \mathcal{E}^2/Z_0$ is the power density per unit area, in W/m^2, with $\mathcal{E}$, in V/m; and $Z_0 = 120\pi$ is the impedance of the free space, in Ω. Knowing that the aperture in m^2 of an isotropic antenna is $A_{\text{iso}} = \lambda^2/4\pi$

$$P_r = \frac{\mathcal{E}^2 \lambda^2}{480\pi^2} = \frac{\mathcal{E}^2 c^2}{480\pi^2 f_c^2} \quad (36)$$

with λ in meters, c in meters/second, and f_c in hertz. Adjusting (36) so as to deal with $\mathcal{E}$ (μV/m) and f_c (MHz)

$$Pr = \frac{(\mathcal{E}\, 10^{-6})^2}{(f_c\, 10^6)^2} \frac{c^2}{480\pi^2} = \frac{1.8998\, 10^{-11} \mathcal{E}^2}{f_c^2} \quad (37)$$

For the isotropic receiver, $10log P_r = 10log P_t + 10log G_t - L_p$. Thus, algebraic manipulation to (37) in logarithm scale yields

$$\underbrace{20 \log \mathcal{E}}_{\text{dB}\mu\text{V/m}} = \underbrace{10log P_t}_{\text{dBW}} + \underbrace{10log G_t}_{\text{dBi}} + \underbrace{20 \log f_c + 107.213 - L_p}_{\text{dB}}. \quad (38)$$

Figure 1. Section of text from journal article.

Tasks:

i. The units of P_r is watts - show that the right hand part of equation 36 has the same units.

ii. Turn equation 36 into log form and then adjust so as to deal with Ɛ in µV/m and f_c in MHz.

iii. Is equation 37 the same as your answer in ii ?

iv. Then turn your equation in ii into equation 38.

[2] Gabriel Porto Villardi; Hiroshi Harada; Fumihide Kojima and Hiroyuki Yano, Primary Contour Prediction Based on Detailed Topographic Data and Its Impact on TV White Space Availability, IEEE Transactions on Antennas and Propagation, 2016, Vol 64, Issue 8, Pages: 3619 - 3631, DOI: 10.1109/TAP.2016.2580164

Answer to Task 1

i. The units of P_r is watts - show that the right hand part of equation 36 has the same units.

I would expect to define the units for each symbol used on the right hand side of the equation and then the answer to fall out of the simplification – not so simple in this case:

$$P_r = \frac{\mathcal{E}^2 \times c^2}{480 \times \pi^2 \times f_c^2} \qquad \text{equation 36}$$

$$[w] = \frac{\left[\frac{V}{m} \times \frac{V}{m}\right] \times \left[\frac{m}{s} \times \frac{m}{s}\right]}{[Hz \times Hz]} \qquad \text{units in place of symbols}$$

Ignore 480 × π^2 as looks like a constant in the form the author has written it down

Replace [Hz] with [S^{-1}] as $frequency = \frac{1}{Time}$ and [Hz] $= \frac{1}{s}$

$$[w] = \frac{\left[\frac{V}{m} \times \frac{V}{m}\right] \times \left[\frac{m}{s} \times \frac{m}{s}\right]}{\left[\frac{1}{s} \times \frac{1}{s}\right]} \qquad \text{units in place of symbols}$$

Simplify to:

$$[w] = [V^2]$$

That is obviously wrong as watts ≠ volts^2.

The problem comes around as the 480 × π which I treated as a constant is made up of Z_o the impedance free space which is 120 × π. The additional 4 × π is the only real constant. The unit of impedance is ohms [Ω] which gives us.

$$[w] = \frac{\left[\frac{V}{m} \times \frac{V}{m}\right] \times \left[\frac{m}{s} \times \frac{m}{s}\right]}{\Omega \times \left[\frac{1}{s} \times \frac{1}{s}\right]} \qquad \text{units in place of symbols}$$

Simplify to:

$$[w] = \left[\frac{V^2}{\Omega}\right]$$

The next trick is to recognise that the right hand side of this equation is one of the ohms law / power formulas from electronics 101. See the Ohm's Law Pie Chart below – this chart uses the symbol R for Ω.

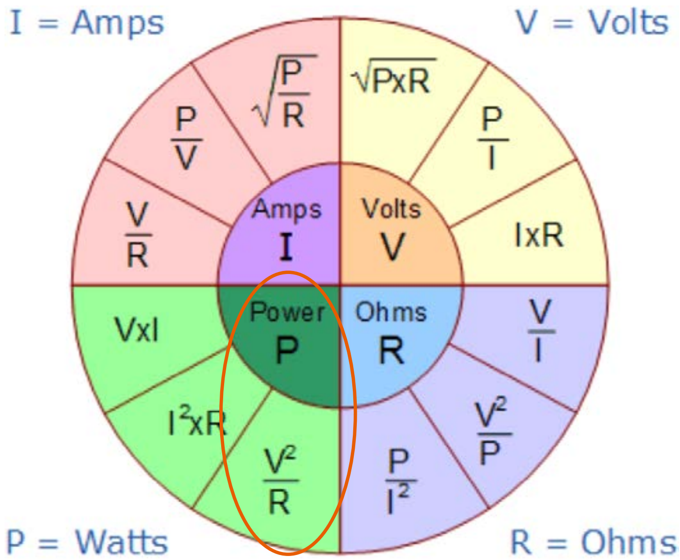

Figure 2. Ohm's law pie chart.

So:

$$\left[\frac{V^2}{\mathrm{R}}\right] = [w]$$

And task 1 is completed.

Answer to Task 2

ii. Turn equation 36 into log form and then adjust so as to deal with Ɛ in μV/m and f_c in MHz then show that your answer is equivalent to equation 37.

$$P_r = \frac{\mathcal{E}^2 \times c^2}{480 \times \pi^2 \times f_c^2} \qquad \text{equation 36}$$

Using everything we have learned about logs:

$$P_r(\mathrm{dBW}) = 10 \cdot \log \mathcal{E}^2 + 10 \cdot \log \mathrm{c}^2 - (10 \cdot \log(480 \times \pi^2) + 10 \cdot \log {f_c}^2)$$

Simplify, gather constants and calculate constants:

$$P_r\,(\mathrm{dBW}) = 20 \cdot \log \mathcal{E} + 132.8 - 20 \cdot \log f_c$$

Where P_r is in [w], Ɛ is in [V/m] and f_c in [Hz]

To get Hz to MHz we need to modify the log equation by subtracting 120 dB (as the frequency variable is in the denominator) which comes from:

$$-20 \cdot \log 1000000 = \; -120\;dB$$

To get from V/m to µV/m we need to modify the log equation by adding -120 dB which comes from:

$$+20 \cdot \log 0.000001 = -100\ dB$$

You add the amount because the Ɛ variable is in the numerator but as it is a negative number you end up adding a negative number which is subtraction.

So we need to subtract a total of 240 dB from our log equation.

$$P_r(\text{dBW}) = 20 \cdot \log \mathcal{E} + 132.8 - 20 \cdot \log f_c - 120 + (-120)$$

$$P_r(\text{dBW}) = 20 \cdot \log \mathcal{E} - 20 \cdot \log f_c - 107.2$$

Answer to Task 3

iii. Is equation 37 the same as your answer in ii ?

$$P_r = \frac{1.8998 \times 10^{-11} \cdot \mathcal{E}^2}{{f_c}^2}$$

Turn into logs

$$P_r(\text{dBw}) = 10 \cdot \log 1.8998 \times 10^{-11} + 10 \cdot \log \mathcal{E}^2 - 10 \cdot \log {f_c}^2$$

$$P_r(\text{dBw}) = -107.2 + 20 \cdot \log \mathcal{E} - 20 \cdot \log f_c$$

Oh yes it is.

Answer to Task 4

iv. Then turn your equation in ii into equation 38.

$$P_r(\text{dBW}) = 20 \cdot \log \mathcal{E} - 20 \cdot \log f_c - 107.2$$

And we are told that

$$P_r(\text{dBW}) = 10 \cdot \log P_t + 10 \cdot \log G_t - L_p$$

Putting the two together

$$10 \cdot \log P_t + 10 \cdot \log G_t - L_p = 20 \cdot \log \mathcal{E} - 20 \cdot \log f_c - 107.2$$

Move $20 \cdot \log \mathcal{E}$ to the left hand side and simplify to get equation 38

$$20 \cdot \log \mathcal{E} = 10 \cdot \log P_t + 10 \cdot \log G_t + 20 \cdot \log f_c - L_p + 107.2$$

Ends…